

1. Bandwidth Calculation

Problem:

A channel has a bandwidth of 3 kHz and a signal-to-noise ratio (SNR) of 20 dB. Calculate the maximum data rate of this channel using the Shannon Capacity formula.

Solution:

The **Shannon Capacity** formula is:

$$C = B \cdot \log_2(1 + SNR)$$

Where:

- C = Channel Capacity (bps)
- B = Bandwidth (Hz)
- SNR = Signal-to-Noise Ratio (linear scale)

1. Convert SNR from dB to linear scale:

$$SNR = 10^{\frac{SNR_{dB}}{10}} = 10^{\frac{20}{10}} = 100$$

2. Substitute into the formula:

$$C = 3000 \cdot \log_2(1 + 100) = 3000 \cdot \log_2(101)$$

3. Approximate $\log_2(101)$:

$$\log_2(101) \approx \frac{\log_{10}(101)}{\log_{10}(2)} = \frac{2.004}{0.301} \approx 6.66$$

4. Calculate C :

$$C = 3000 \cdot 6.66 \approx 19,980 \text{ bps}$$

Answer: The maximum data rate is approximately **19.98 kbps**.

2. Nyquist Theorem

Problem:

A noiseless channel has a bandwidth of 5 MHz. If four signal levels are used, calculate the maximum data rate.

Solution:

The Nyquist Formula is:

$$C = 2 \cdot B \cdot \log_2(L)$$

Where:

- C = Data Rate (bps)
- B = Bandwidth (Hz)
- L = Number of signal levels

1. Substitute the given values:

$$C = 2 \cdot 5 \cdot 10^6 \cdot \log_2(4)$$

2. Calculate $\log_2(4)$:

$$\log_2(4) = 2$$

3. Calculate C :

$$C = 2 \cdot 5 \cdot 10^6 \cdot 2 = 20 \cdot 10^6 = 20 \text{ Mbps}$$

Answer: The maximum data rate is **20 Mbps**.

3. Signal-to-Noise Ratio (SNR)

Problem:

A communication channel has a bandwidth of 2 kHz and a maximum data rate of 16 kbps. What is the SNR (in dB)?

Solution:

Rearranging the Shannon Capacity formula:

$$SNR = 2^{\frac{C}{B}} - 1$$

Where:

- C = Data Rate (bps)
- B = Bandwidth (Hz)

1. Substitute the given values:

$$SNR = 2^{\frac{16000}{2000}} - 1 = 2^8 - 1 = 256 - 1 = 255$$

2. Convert to dB:

$$SNR_{dB} = 10 \cdot \log_{10}(SNR) = 10 \cdot \log_{10}(255)$$

3. Approximate $\log_{10}(255)$:

$$\log_{10}(255) \approx 2.406$$

4. Calculate SNR_{dB} :

$$SNR_{dB} = 10 \cdot 2.406 \approx 24.06 \text{ dB}$$

Answer: The SNR is approximately **24.06 dB**.

4. Time to Transmit a File

Problem:

A file of size 100 MB needs to be transmitted over a channel with a data rate of 10 Mbps. Calculate the transmission time.

Solution:

The formula for transmission time is:

$$T = \frac{\text{File Size}}{\text{Data Rate}}$$

1. Convert file size to bits:

$$100 \text{ MB} = 100 \cdot 8 \cdot 10^6 = 800 \cdot 10^6 \text{ bits}$$

2. Convert data rate to bits per second:

$$10 \text{ Mbps} = 10 \cdot 10^6 \text{ bps}$$

3. Calculate transmission time:

$$T = \frac{800 \cdot 10^6}{10 \cdot 10^6} = 80 \text{ seconds}$$

Answer: The transmission time is **80 seconds**.

5. Bit Error Rate (BER)

Problem:

A digital communication system transmits data at 1 Mbps with a BER of 10^{-6} . How many errors are expected in one hour of transmission?

Solution:

The number of errors can be calculated as:

$$\text{Errors} = \text{Data Rate} \cdot \text{Time} \cdot \text{BER}$$

1. Convert time to seconds:

$$1 \text{ hour} = 3600 \text{ seconds}$$

2. Substitute the values:

$$\text{Errors} = 1 \cdot 10^6 \cdot 3600 \cdot 10^{-6}$$

3. Simplify:

$$\text{Errors} = 3600$$

Answer: The system is expected to encounter **3600 errors** in one hour.

6. Propagation Delay

Problem:

A signal travels through a 500 km link at a speed of 2×10^8 m/s. Calculate the propagation delay.

Solution:

The formula for propagation delay is:

$$\text{Propagation Delay} = \frac{\text{Distance}}{\text{Propagation Speed}}$$

1. Convert distance to meters:

$$500 \text{ km} = 500 \cdot 1000 = 500,000 \text{ m}$$

2. Substitute values:

$$\text{Propagation Delay} = \frac{500,000}{2 \times 10^8} = 2.5 \times 10^{-3} \text{ seconds}$$

3. Convert to milliseconds:

$$\text{Propagation Delay} = 2.5 \text{ ms}$$

Answer: The propagation delay is 2.5 ms.

7. Transmission Time vs. Propagation Time

Problem:

A file of size 2 MB is transmitted over a link with a bandwidth of 10 Mbps. The distance between the sender and receiver is 1,000 km, and the propagation speed is 2×10^8 m/s. Calculate:

1. Transmission Time
2. Propagation Time

Solution:

1. Transmission Time

$$\text{Transmission Time} = \frac{\text{File Size}}{\text{Data Rate}}$$

Convert file size to bits:

$$2 \text{ MB} = 2 \cdot 8 \cdot 10^6 = 16 \cdot 10^6 \text{ bits}$$